PG – 272

II Semester M.Sc. Degree Examination, June/July 2014
(NS) (2006 Scheme)
MATHEMATICS
M - 204 : Partial Differential Equations

Time : 3 Hours   Max. Marks : 80

Instructions : *1) Answer* ***any five*** *questions choosing at least* ***two*** *from* ***each*** *Part.*
2) ***All*** *questions carry* ***equal*** *marks.*

PART – A

1. a) Define linear, semilinear, quasilinear and nonlinear equations of first order partial differential equation with an example each. Explain the method of characteristics for solving quasilinear partial differential equation.

$a(x, y, u)\, u_x + b(x, y, u)\, u_y = c(x, y, u)$ **6**

b) Solve the following by the method of characteristics

i) $u_x + u_y + u = 1$ with $u = \sin x$ on $y = x + x^2$

ii) $u \cdot u_x + u_y = 1$ with $u = 0$ on $y^2 = 2x$. **10**

2. a) Solve the Cauchy problem $yu_x + xu_y = u$

With the Cauchy data

$u(x, 0) = x^3,\ u(0, y) = y^3$ **5**

b) Solve the initial value problem

$u_t + u.u_x = 0,\ x \in R,\ t > 0$

$$u(x,0) = \begin{cases} a^2 - x^2, & |x| \le a \\ 0, & |x| \ge a \end{cases}$$ **6**

c) Solve the problem

$p^2 x + qy - u = 0$ with $u = -x$ on $y = 1$ **5**

P.T.O.

3. a) Solve :

i) $\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} - \frac{\partial^3 z}{\partial x \partial y^2} + 2\frac{\partial^3 z}{\partial y^3} = e^{x+y}$

ii) $(D^3 + D^2D' - DD'^2 - D'^3)z = e^x \cos 2y$ **6**

b) Classify the equation :

$$\frac{\partial^2 z}{\partial x^2} + x^2 \frac{\partial^2 z}{\partial y^2} = 0$$

and reduce it to its canonical form. **5**

c) Solve by Monges method

$rq^2 - 2pqs + tp^2 = pt - qs$ **5**

4. a) Explain the method of solving the equation

$Rr + Ss + Tt + U(rt - s^2) = V$

Where R, S, T, U, V are functions of x, y, z, p and q. **6**

b) Write down the necessary condition for the extremization of the functional

$$I[u(x,y)] = \int_{x=a}^{b} \int_{y=c}^{d} f(x, y, u, u_x, u_y)\, dx\, dy.$$

Using this result make a weak formulation of $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0.$ **5**

c) Verify whether

$x^3 u_{xx} + (y^2 + yz) u_{yy} + (x+y)^2 u_{zz} + 3x^2 u_x + (2y + z) u_y = 0$ is self adjoint or not. **5**

PART – B

5. a) Solve the Neumann problem for a circle. **8**

b) Show that a separable solution of the Laplace's equation in cylindrical polar coordinates yields a Bessel differential equation. **8**

6. a) Using an appropriate Fourier transform obtain the D′ Alembert solution of the Cauchy problem involving a one dimensional wave equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}, \; -\infty < x,\; t < \infty$$ **8**

Subject to $\left.\begin{array}{l} u(x,0) = f(x) \\ \dfrac{\partial u}{\partial t}(x,0) = g(x) \end{array}\right\} -\infty < x < \infty$

b) Solve $\dfrac{\partial^2 u}{\partial t^2} = \dfrac{\partial^2 u}{\partial x^2} = x^2,\ 0 \le x \le 1, t \in R$

Subject to $\left.\begin{array}{l} u(x,0) = x \\ \dfrac{\partial u}{\partial t}(x,0) = 0 \end{array}\right\}, 0 \le x \le 1$

$\left.\begin{array}{l} u(0,t) = 1 \\ u(1,t) = 0 \end{array}\right\}, t \in R$ **8**

7. a) Using Duhamel's principle obtain the solution of the IBVP. **8**

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2},\ 0 \le x \le 1,\ t \ge 0$$

$u(x, 0) = 0,\ \ 0 \le x \le 1$

$\left.\begin{array}{l} u(0,t) = 1 \\ u(1,t) = 0 \end{array}\right\}, t \ge 0$

b) Illustrate Green's function approach for a simple parabolic partial differential equation. **8**

8. a) Using a suitable partial differential equation with two independent variables explain the concept of a similarity transformation and hence obtain the resulting ordinary differential equation. **8**

b) Obtain a series solution in x and y of the

BVP : $xu_x + u_y = 3u^2$;

$u(x, 0) = x^2$, $u(0, y) = 0$. **8**